Optimization of Propeller Blade Twist by an Analytical Method

Li Ko Chang* and John P. Sullivan†
Purdue University, West Lafayette, Indiana

Based on the vortex lattice method and nonlinear programming, the problem for predicting optimum propeller blade twist has been formulated and solved to maximize the propulsive efficiency under the constraint of constant power consumption. The propeller is represented by a curved lifting line and a number of control points. The optimum twist distribution can be determined for a specified geometry of the lifting line. The method can be applied to complex blade shapes (swept, bent, propletted, and biblades). To demonstrate the method, the geometry of the lifting line of a straight blade and a propletted blade has been employed. The twist distribution and the ideal efficiency for the optimized and unoptimized blades are compared. The predicted improvement in ideal efficiency is about 1-6% for the optimized blade with proplet over that of the original propeller.

Introduction

THE selection of propeller blade shapes in aerodynamic design has a direct influence on efficiency. High-performance propellers can be obtained using optimum blade shapes, which can be determined by an optimization method. This method is employed to maximize the propulsive efficiency under the requirement of constant power consumption.

In 1942, Lock et al.[1] worked on the optimum pitch distribution of a propeller using the method of calculus of variations. Later Haines and Diprose[2] used the same method to optimize the twist and chord distribution of a propeller. These approaches were used for straight blades and the results were shown in Ref. 2. Moriya[3] also worked on finding the best twist distribution for a straight-blade propeller using the condition that the first derivatives of the objective function vanish at stationary points. This approach needs the input of tabulated data and is also limited to the treatment of straight blades.

The purpose of this paper is to present an optimization method that can be employed to predict the optimum twist distribution of the propeller blade using the technique of nonlinear programming. The method is capable of predicting optimum twist for both simple and complex blade shapes (straight, swept, bent, propletted, and biblades).

Formulation

The aerodynamic performance analysis employed in the optimization method is based on Refs. 4 and 5. An idealized model is assumed for the analysis method since the viscous drag and compressible effects have not been included to conserve computer time. As shown in Fig. 1, the propeller is modeled by a bound vortex and a number of control points. The bound vortex has a varying vortex strength, which causes the vorticity to be shed and form a helical vortex sheet. The strengths of the bound vortex elements are determined by requiring the velocity at the control points to satisfy the boundary condition. The performance of the propeller is calculated after the influence coefficients and the induced velocities have been determined.

In the optimization analysis, the position of the curved lifting line and the chord length distribution are assumed to be given. The design vector to be solved is the optimum twist distribution that gives maximum propulsive efficiency (objective function). The nonlinear programming problem to be solved can be stated as follows:

Minimize

$$f(x) \text{ subject to } C_p(x) - C_{pa} = 0, \; x \geq 0 \tag{1}$$

where

$$x = [x_1, x_2, \ldots x_n]^T$$

$f(x) = -$(ideal efficiency)
$C_p(x) =$ the current value of the power coefficient, calculated in the optimization process
C_{pa} = design power coefficient
$x =$ design vector (the angles of twist of the vortex lattice elements)
$n =$ number of vortex lattice elements

The objective function $f(x)$ is calculated using the computer program for the performance analysis of propellers.[4,5] Comparisons between experimental data and the predicted results for the NASA advanced turboprop are given in Ref. 6. Although an ideal model has been assumed in this paper, the optimization method can be extended to include the neglected factors. Note that all quantities involved in the performance analysis and the optimization are of dimensionless forms.

Optimization Method

The problem formulated in Eq. (1) was solved using the computer code for the combination of the performance analysis and the penalty function optimization method.[7,8] There are numerous methods[9] that can be selected to solve a nonlinear programming problem. The penalty function method of Fiacco and McCormick[7] was selected because of its high reliability. In performing the optimization, the C_{pa} in the equality constraint is the design power coefficient during cruise. The penalty function method transforms Eq. (1) into an alternative form, which consists of the original objective function and penalty terms [Eq. (2)]. The problem is solved by solving a sequence of unconstrained minimization problems.

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*Aerospace Research Engineer, School of Aeronautics and Astronautics. Member AIAA.
†Associate Professor, School of Aeronautics and Astronautics. Member AIAA.

The penalty function is given as

$$P(x,r) = f(x) + \frac{1}{r}\sum_{i=1}^{m} h_i^2(x) - r\sum_{i=1}^{k} \ell n g_i(x) \tag{2}$$

where

$p(x,r)$ = penalty function
r = penalty parameter, which is a positive monotonically decreasing sequence, $r^j > r^{j+1}$
$f(x)$ = objective function
$h_i(x)$ = equality constraints
$g_i(x)$ = inequality constraints
m = number of equality constraints
k = number of inequality constraints

The minimization starts from an initial blade twist x^0 and approaches to the minimum according to the relation

$$x^{j+1} = x^j + \alpha^j s^j$$

where x^j is the blade twist corresponding to a current minimum of $P(x,r)$ and x^{j+1} a new blade twist that gives a new minimum point along the search direction s^j, with the optimal step length α^j. The direction vector is determined by Davidon-Fletcher-Powell's method,[8,10,11]

$$s^j = -A^j \nabla P(x^j, r^j)$$

where A^j is a symmetric positive definite $n \times n$ matrix and $\nabla P(x^j, r^j)$ the gradient of the penalty function. Matrix A^j is updated by

$$A^{j+1} = A^j + \frac{(\Delta x^j)(\Delta x^j)^T}{(\Delta x^j)^T(\Delta g^j)} - \frac{A^j(\Delta g^j)(\Delta g^j)^T(A^j)}{(\Delta g^j)^T A^j(\Delta g^j)}$$

At the starting point $j=0$, A^0 is approximated by the identity matrix of rank n. Δx^j is the difference between x^{j+1} and x^j, Δg^j the difference of gradients at x^{j+1} and x^j and T the transpose of the matrix. The optimal step length to move along s^j is obtained by the golden section method.[8] The algorithm runs until the convergence criteria are satisfied.

Description of the Optimization Process

The optimization algorithm performs both the optimization and the evaluation of the objective function. A general description of the algorithm is:

1) Read input data for the objective function and the optimization process. Define design variables and constraints.
2) Calculate the influence coefficients.
3) Start the optimization process with current input data.
4) Update the position of the control points, twist angles, and circulations.
5) Update the induced velocities, thrust coefficient, power coefficient, and objective function.
6) Check the condition of convergence. If it is satisfied, stop; if not satisfied, make a new step of movement and go to step 3.

Propeller Description

The optimization method has been applied to two different kinds of propellers.[12] The characteristics of these propellers are:

1) Fixed pitch, low-speed cruise (58 m/s) general aviation propeller (denoted by GA). The cruise advance ratio is 0.782 and the angle of twist at three-quarter radius is 21.2 deg.
2) Variable pitch, high-speed cruise (134 m/s) general aviation propeller, called the Purdue model (denoted by PM). The cruise advance ratio is 2.2 and the angle of twist at 0.75 radius ratio is 45.4 deg.

Propletted blades for both GA and PM propellers have also been analyzed using the optimization method. A proplet is a blade tip device to improve the efficiency by reducing the induced drag. Comparison of the aerodynamic performance for propellers with and without proplets is given in Refs. 4 and 12. A proplet is specified by the proplet height h/R and

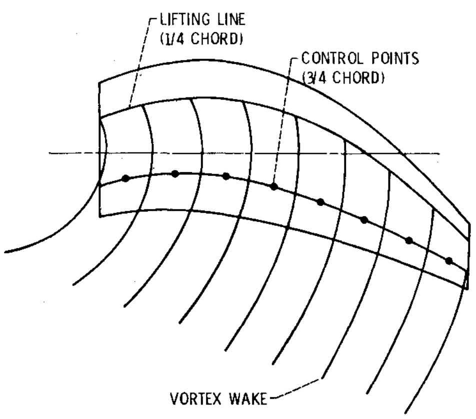

Fig. 1 Vortex lattice elements of a propeller.

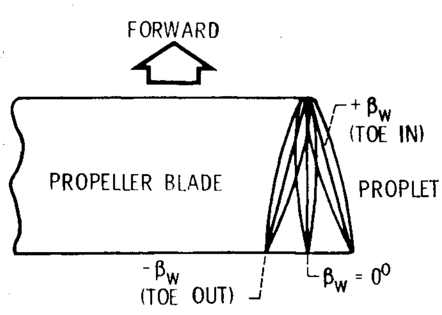

Fig. 2 Proplet angle description.

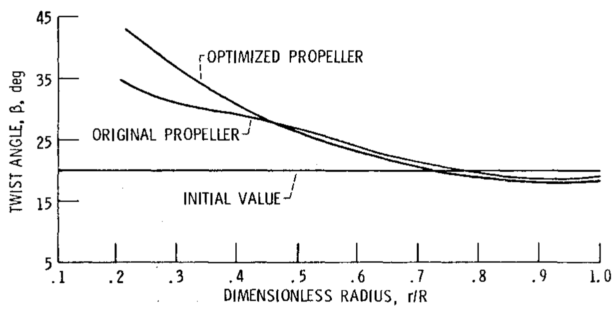

Fig. 3 Comparison of optimized and unoptimized angle of twist of the GA propeller.

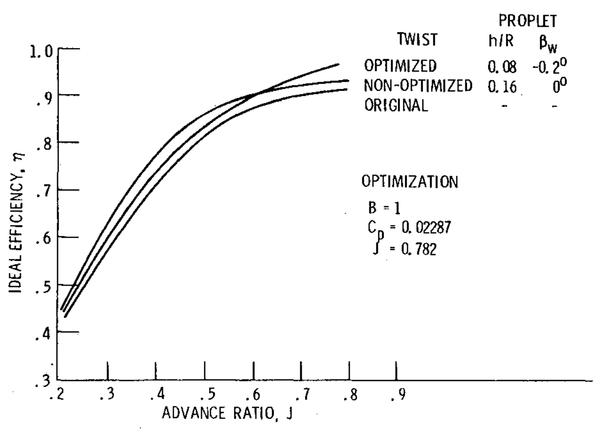

Fig. 4 Comparison of optimized and unoptimized efficiency of the GA propeller.

the proplet angle β_w, see Fig. 2. The proplet height for GA and PM blade was not considered to be a design variable since the improvement in efficiency is almost linearly proportional to the proplet height. Note that the optimization is based upon one blade ($B=1$).

Results

GA Propeller

The initial twist distribution assigned to the optimization process is a constant twist of 0.8 rad. Other infeasible values could also be used for this method. The geometry for the lifting line and the chord distribution of the original propeller were used. The optimization was performed at the design power coefficient of 0.02287 and at the cruise advance ratio of 0.782. Figure 3 presents the effect of optimization on blade twist, which is plotted vs the dimensionless radius of the blade. The twist distributions of the optimized blade and the original propeller are about the same from 0.45 radius ratio to the blade tip. A significant difference appears inboard to the blade. This difference may have resulted from the nacelle effect, which may be taken into account for the original propeller, but the analysis method is dealing with an open propeller.

The comparison of ideal efficiency is shown in Fig. 4 for three different propeller geometries: 1) the original propeller, 2) a nonoptimized original propeller with a proplet (proplet height $=0.16$ and $\beta_w=0.0$ deg), and 3) an optimized propeller with a proplet (proplet height $=0.08$ and $\beta_w=-0.2$ deg). The ideal efficiency for cases 1 and 2 was calculated using the twist of the original design. The efficiency for case 3 was calculated using the optimized twist. As demonstrated in Fig. 4, the efficiency improvement of the optimized propeller is about 6% greater than the original propeller and 3% greater than the propletted blade (case 2). Note that the proplet height for cases 2 and 3 are not the same. A slightly larger improvement would be obtained if the proplet height of case 2 were halved.

PM Propeller

Figure 5 presents the comparison of the twist distribution for the optimized blade and the original propeller. The optimization was performed at an advance ratio of 2.2 and with a blade twist of 45.4 deg at the three-quarter radius. The optimized twist shows a similar distribution to that of the original propeller, but the optimization causes the twist to increase inboard and decrease outboard. Two inequality constraints: 1) $\beta_w \geq 0$ deg, and 2) $\beta_w \leq 0$ deg have been individually imposed in the optimization process. Figure 6 shows the twist distribution of case 1 and Fig. 7 for case 2. The results for a propletted blade and the original propeller without proplet are given in the figures. An efficiency improvement is obtained only for case 2 ($\beta_w=-5.48$ deg), which is referred to as a toe-out proplet. This result was described in Ref. 12, where the proplet angle was varied in order to get the best proplet angle, leaving the twist unchanged.

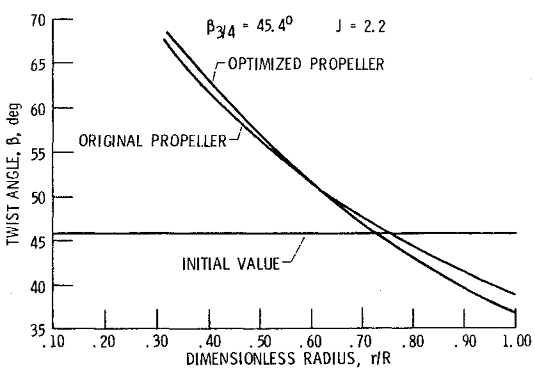

Fig. 5 Comparison of optimized an unoptimized angle of twist of the PM.

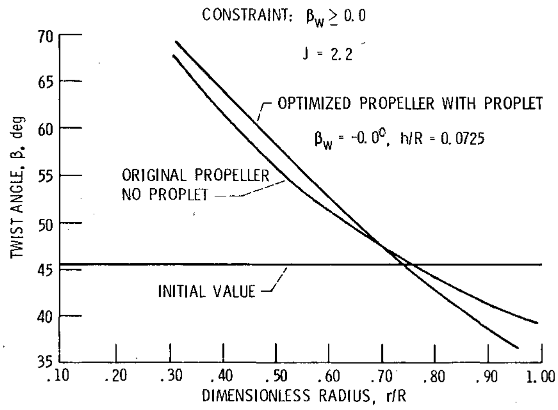

Fig. 6 Comparison of optimized and unoptimized angle of twist of the PM with constraint $\beta_w > 0$ deg.

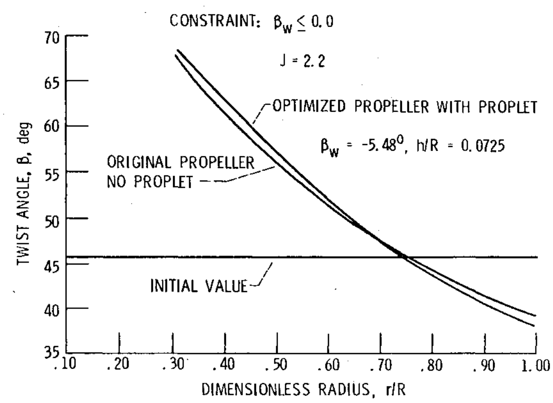

Fig. 7 Comparison of optimized and unoptimized angle of twist of the PM with constraint $\beta_w < 0$ deg.

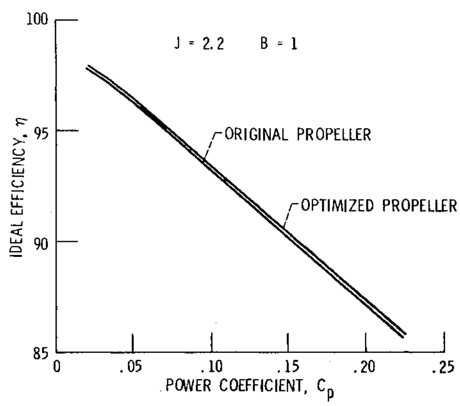

Fig. 8 Comparison of optimized and unoptimized propeller efficiency for the PM without proplet at different power coefficients.

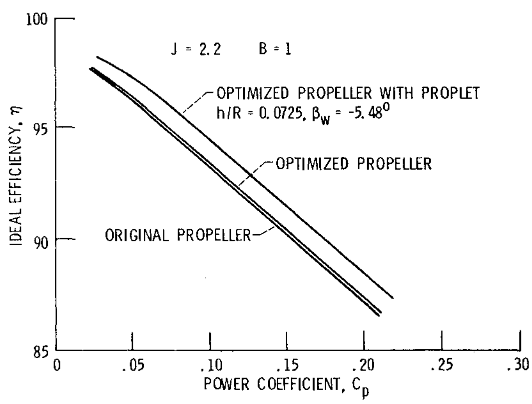

Fig. 9 Comparison of optimized and unoptimized propeller efficiency for the PM with proplet at different power coefficients.

A comparison of ideal efficiency for the optimized propeller and the original blade is presented in Fig. 8. The contribution made by the PM blade is small. This indicates that the original design is good. The efficiency improvement with a propletted blade is given in Fig. 9. The gain in efficiency for an optimzed propeller with a toe-out proplet over the original propeller is about 1%.

The difference in the aerodynamic performance of propellers as calculated by Goldstein's method[13] and the optimization was considered. In Ref. 4, a study was made of the ideal efficiency as calculated by Goldstein's method and the vortex lattice method (the performance analysis code) using NACA 109622 and NASA SR-2 straight-blade propellers. It has been shown that there is no appreciable difference in the ideal efficiency obtained by the Goldstein's approach and the vortex lattice method. Thus, the ideal performance presented in this paper for an unoptimized straight blade, calculated by the performance analysis code, is consistent with the ideal performance calculated using the Goldstein's method.

Conclusions

This paper has presented a method for the optimization of propeller twist. Optimum blade twist can be predicted by specifying the geometry of the lifting line and the desired chord length of the blade. Various optimum blade twists can be explored by simply selecting different geometries of the lifting line. This method can be used for the aerodynamic design of high-performance propeller blade shapes. The proposed method has been applied to the general aviation propellers, with and without proplet. Ideal efficiency improvements up to 6% were obtained for an optimized propeller with proplets compared with the original propeller.

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